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Do quark mass effects survive in the high-Q^2 limit of DIS?

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Abstract. Quark mass effects are analyzed at high Q^2 in the current fragmentation region of DIS. It is found that the linear combination $F_2 - 2.75F_2^c$ scales at large Q^2 and small x. We obtained a lower bound for the ratio F_2^c/F_2 which lies very close to the data from HERA.

1 Introduction

The data on open-charm production in deep inelastic scattering (DIS) from the HERA collider [1,2] show that its contribution to the structure function, F_2^c, runs up to 40% at the measured values of x and Q^2 and increases faster than F_2 when x becomes smaller. The contribution of the b-quarks, F_2^b, to the total structure function is of the order 2–3%, as one can see from recent measurements of open-beauty production [3].

One often believes that, with increasing energy of the colliding particles, W, and momentum transfer squared, Q^2, the mass effects become insignificant. However, arguments were given in [4,5] that the difference between the DIS structure functions with open heavy quark production in the current fragmentation region and structure functions of the process without heavy flavors scales for large Q^2, i.e. it depends on the Bjorken variable x and the heavy quark mass m_Q only. This result gives an opportunity to obtain model independent (i.e. independent on the concrete choice of the gluon distribution inside the nucleon) lower bound for F_2^c, which is in an agreement with the experimental data on F_2^c [4,5].

In the present work we analyze the problem of the mass scale influence on the behavior of the physical quantities in the framework of the operator product expansion [6].

It is convenient to introduce the quantity

$$F_2^{Q\bar{Q}} = F_2^Q / e_Q^2 \tag{1}$$

instead of F_2^Q $(Q = c, b)$. Analogously for the light quarks $q = u, d, s$ (that we treat as massless), we define

$$F_2^{q\bar{q}} = F_2^q / e_q^2, \tag{2}$$

where $e_{Q(q)}$ are the quarks' electric charges.

The operator product expansion gives the following expression for the $F_2^{Q\bar{Q}}$:

$$\begin{aligned}
&\frac{1}{x} F_2^{Q\bar{Q}}(x, Q^2, m_Q^2) \\
&= C_g\left(\frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_g(\mu^2)[x] \\
&+ C_Q\left(\frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_Q(\mu^2)[x] \\
&+ C_q\left(\frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_q(\mu^2)[x].
\end{aligned} \tag{3}$$

In (3) the quantities C_i are coefficient functions, f_i are matrix elements of the corresponding composite operators, which can be identified with distributions of quarks and gluons inside the nucleon, and μ is the renormalization scale of the composite operators. The symbol $\otimes$ means convolution in the variable x,

$$a \otimes b[x] = \int_x^1 \frac{\mathrm{d}y}{y} a(y) b\left(\frac{x}{y}\right). \tag{4}$$

In (3) we have neglected higher twists contributions. To simplify the notation, we do not show below the α_s-dependence of the coefficient functions.

It is well known that C_i and f_i separately depend on the renormalization scheme. Usually, the $\overline{\mathrm{MS}}$-scheme is more preferable than the MOM-scheme, because of more complicated calculations in the framework of the latter.

Nevertheless, the MOM-scheme has some advantages. One of them is the universality of the calculational algorithm for the coefficient functions in any order of perturbation theory. Advantages of use of the scheme with momentum subtraction in the case of heavy quarks were noted in [7], where the authors proposed a mixed, so-called CWZ renormalization scheme.

The purpose of the present work is to show that the effects related to the heavy quark mass m_Q, "survive" in the large-Q^2 limit. There is a linear combination of the structure functions F_2 and F_2^c that has scaling properties.

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It appears that this scaling takes place in different renormalization schemes. The lower bound for the ratio F_2^c/F_2 as a function of x at fixed values of Q^2 is also calculated. The results are compared with the experimental data.

2 Asymptotic relations between structure functions

We are interested in the behavior of F_2^c at large Q^2 and small x. We suppose that the production of heavy quarks in this region results from gluons, and heavy quarks are not included in the evolution of light quarks and gluons.

So we can write

$$\frac{1}{x}F_2^{Q\bar{Q}}(x,Q^2,m_Q^2) = C_g\left(\frac{Q^2}{\mu^2},\frac{m_Q^2}{\mu^2}\right)\otimes f_g(\mu^2)[x] + C_Q\left(\frac{Q^2}{\mu^2},\frac{m_Q^2}{\mu^2}\right)\otimes f_Q(\mu^2)[x]. \quad (5)$$

Taking $\mu^2=\mu_0^2$, where $\Lambda^2_{\rm QCD}\ll\mu_0^2\ll Q^2$, and neglecting "intrinsic charm" ("beauty") inside the nucleon, i.e. assuming that there is a scale μ_0^2 at which the heavy quark distribution function is small as compared to the gluon one, we simplify (5) to the relation

$$\frac{1}{x}F_2^{Q\bar{Q}}(x,Q^2,m_Q^2) = C_g\left(\frac{Q^2}{\mu_0^2},\frac{m_Q^2}{\mu_0^2}\right)\otimes f_g(\mu_0^2)[x]. \quad (6)$$

Let us now define the quantity

$$\Delta F_2 = F_2^{q\bar{q}}(x,Q^2) - F_2^{Q\bar{Q}}(x,Q^2,m_Q^2). \quad (7)$$

Assuming that at small x the main contribution to the DIS structure function without heavy quark production is determined by gluons, i.e. by the formula analogous to (6), we find

$$\frac{1}{x}\Delta F_2 = \Delta C_g\otimes f_g(\mu_0^2)[x], \quad (8)$$

where

$$\Delta C_g = C_g\left(y,\frac{Q^2}{\mu_0^2},0\right) - C_g\left(y,\frac{Q^2}{\mu_0^2},\frac{m_Q^2}{\mu_0^2}\right). \quad (9)$$

Calculations of the gluon coefficient function in the order $O(\alpha_{\rm s})$ in the MOM-scheme were given in [8]. Using the expression obtained there, we get from (9)

$$\Delta C_g \simeq \Delta C_g^{(1)}\left(y,\frac{m_Q^2}{\mu_0^2}\right) = \frac{\alpha_{\rm s}}{8\pi}\left\{\left(y^2+(1-y)^2\right)\ln\left[1+\frac{m_Q^2}{\mu_0^2 y(1-y)}\right] - \frac{m_Q^2\left(1+2y(1-y)\right)}{m_Q^2+\mu_0^2 y(1-y)}\right\}. \quad (10)$$

Thus, the quantity ΔF_2 tends to the finite limit $\Delta F_2(x, m_Q^2)$ at large Q^2. It was shown in [8], that this result is not an artifact of the MOM-scheme, although the very expression for the quantity ΔC_g depends, of course, on the renormalization scheme.

Starting from the expression for ΔC_g, (10), it is easy to see that for $y\le 0.1$ (the small y region is most important in the analysis of the behavior of the structure function at $x\ll 1$)

$$\Delta C_g > 0. \quad (11)$$

It follows from the explicit form of $C_g\left(y,Q^2/\mu^2\right)$ in the MOM-scheme for the massless case in the order $O(\alpha_{\rm s})$ for any Q^2 and μ^2 that [8]

$$C_g\left(y,\frac{Q^2}{\mu_0^2}\right)\Big|_{Q^2=m_Q^2} > \Delta C_g\left(y,\frac{m_Q^2}{\mu_0^2}\right). \quad (12)$$

Then from (8), (11) and (12) we conclude

$$F_2^{q\bar{q}}(x,Q^2)\big|_{Q^2=m_Q^2} > \Delta F_2(x,m_Q^2) > 0. \quad (13)$$

The inequalities (13) are used below to obtain the lower bound for the ratio F_2^c/F_2.

3 Charm contribution to the structure function

It has been found in the previous section (see (8) and (10)), that the difference between the contributions of light and heavy flavors to the DIS structure function scales for $Q^2\to\infty$.

Taking this result as a basis, it is easy to see that the following linear combination [5] has scaling behavior, with α being an arbitrary constant:

$$\Sigma_\alpha(x,Q^2) \equiv F_2(x,Q^2) + \alpha F_2^c(x,Q^2,m_c^2) - (4\alpha+11)F_2^b(x,Q^2,m_b^2). \quad (14)$$

To cancel the contribution of the b-quarks from (14), we have taken $\alpha=-2.75$. Then we obtain the prediction that the linear combination

$$\Sigma = F_2 - 2.75F_2^c \quad (15)$$

must tend to a function that depends only on the Bjorken variable x (and the heavy quark mass) for $Q^2\to\infty$.

Using the expression for ΔC_g in the leading order in $\alpha_{\rm s}$, we find that the above difference (15) tends to the scaling limit in the region $m_Q^2\ll Q^2$ in the following way:

$$\frac{1}{x}\Sigma = \frac{1}{9}\left[7\Delta C_g^{(1)}\left(\frac{m_c^2}{\mu_0^2}\right) - \Delta C_g^{(1)}\left(\frac{m_b^2}{\mu_0^2}\right)\right] \otimes f_g(\mu_0^2)[x] + \frac{m_b^2-7m_c^2}{Q^2}\ln\left(\frac{Q^2}{\mu_0^2}\right)\cdot h\otimes f_g(\mu_0^2)[x], \quad (16)$$

where

$$h(y) = \frac{1}{9}y(1-y)[(2-3y)^2+3y^2]. \quad (17)$$

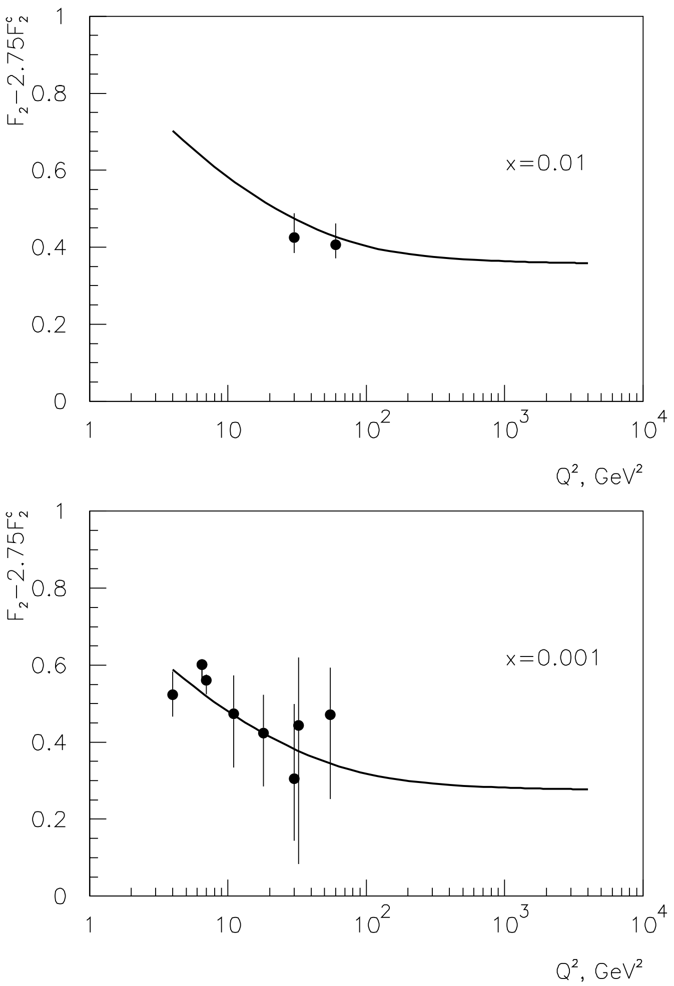

Fig. 1. Q^2-dependence of the difference of structure functions at two fixed values of x. Solid curves are obtained by fitting the data on F_2^c from [2,9]. The experimental points have x closest to the given values $x = 0.01$ and $x = 0.001$

Since $m_b^2 - 7m_c^2 > 0$, we conclude that the correction in the expression for $\Sigma(x, Q^2)$, (16), is positive and decreasing with Q^2.

To compare our results with the recent experimental data, we parametrize F_2^c according to the expression (16) for $\Sigma(x, Q^2, m_Q^2)$ and fit the data from the HERA collider [2].

For F_2 we use the parametrization of the H1 collaboration [1]:

$$F_2(x, Q^2) = \left[ax^b + cx^d(1 + e\sqrt{x}) \times \left(\ln Q^2 + f \ln^2 Q^2 + \frac{h}{Q^2}\right)\right](1 - x)^g,$$

with parameters defined in Table 1.

For F_2^c we choose the expression qualitatively coherent to the asymptotic behavior of the quantity $\Sigma = F_2 - 2.75F_2^c$ with the variable Q^2 (16)

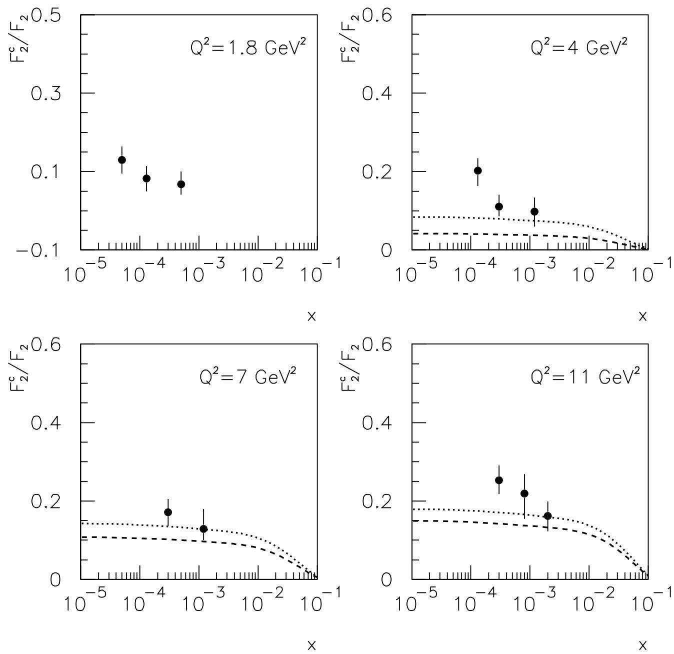

Fig. 2. The ratio F_2^c/F_2 as a function of the variable x at fixed values of Q^2. Dashed curves represent the lower bound for F_2^c/F_2 for the c-quark mass $m_c = 1.7\,\mathrm{GeV}$, dotted curves correspond to $m_c = 1.3\,\mathrm{GeV}$. Experimental points are taken from [2]

Table 1. Values of parameters in the expression for F_2 (18)

a	b	c	d
3.1	0.76	0.124	−0.188
e	f	g	h
−2.91	−0.043	3.69	$1.4\,\mathrm{GeV}^2$

Table 2. Values of parameters in the expression for F_2^c (18)

$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{g}$	$\bar{h}$
0.28	0.15	−0.08	5.00	$1.86\,\mathrm{GeV}^2$

$$F_2^c(x, Q^2) = \frac{1}{2.75} F_2(x, Q^2) - \bar{a}x^{\bar{b}}(1 - x)^{\bar{g}} \left[1 + x^{\bar{c}} \frac{\bar{h}}{Q^2} \ln Q^2\right], \quad (18)$$

where F_2 is defined above (18). Fitting the data from HERA [2] for $6.5\,\mathrm{GeV}^2 \le Q^2 \le 130\,\mathrm{GeV}^2$ gives the values of the parameters in Table 2 and $\chi^2/\mathrm{n.d.f.} = 34.6/36 = 0.96$.

In Fig. 1 we show the dependence of the quantity Σ on Q^2 for two values of x, for which we have the array of experimental data, obtained at different Q^2 and x, closest to the given values $x = 0.01$ and $x = 0.001$. As we see, the experimental data are in good agreement with our result on the approach to the scaling behavior from above, but the very existence of the scaling may be checked at higher Q^2.

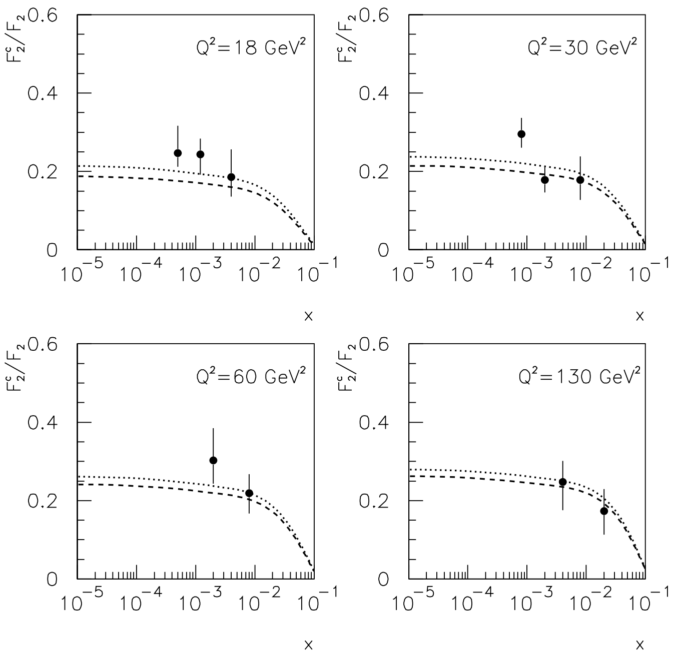

Fig. 3. The same as in Fig. 2, but for other values of Q^2

As has been found in [5], the inequalities (13) allow us to obtain the following estimation for the ratio of the structure functions:

$$\frac{F_2^c(x,Q^2)}{F_2(x,Q^2)} > 0.4\left(1-\frac{F_2(x,m_c^2)}{F_2(x,Q^2)}\right). \qquad (19)$$

It is important to stress that we did not use any parametrization for F_2^c to obtain the inequality, and also that it does not depend on the behavior of the gluon distribution.

The curves represented in Figs. 2 and 3 are calculated by using the formula (19) for two values of the c-quark mass and are compared with the data of the ZEUS collaboration [2]. Despite the fact that these curves are only lower bounds for the ratio F_2^c/F_2, they lie very close to the experimental points.

Our estimations show that lower-bound curves are also in a good agreement with new preliminary data of the ZEUS collaboration [9], including the data for the maximal measured value $Q^2 = 565\,\mathrm{GeV}^2$.

4 Conclusions

In the present work we analyzed quark mass effects in DIS with the help of OPE. It is shown by calculating in the leading order that the new scaling takes place in DIS; a certain linear combination of the DIS structure function and the DIS structure function with open-charm production scales in the limit of large Q^2.

It is found that this specific scaling is in a good agreement with the Q^2 trend of the recent experimental data on F_2^c and F_2 obtained at HERA.

Certainly, higher orders could change the theoretial conclusion about such a scaling. At present the first order result can serve as an indication of the existence of an interesting physical phenomenon. The influence of higher orders is a subject of our further work.

We also calculated the lower bound for the ratio F_2^c/F_2 as a function of x at fixed Q^2, independent on the shape of the gluon distribution in the nucleon, and compared it with the data from ZEUS collaboration. This lower bound appears to be quite close to the data.

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References

1. S. Aid et al., H1 Collab., Nucl. Phys. B **470**, 3 (1996)
2. J. Breitweg et al., ZEUS Collab., Eur. Phys. J. C **12**, 35 (2000)
3. C. Adloff et al. H1 Collab., Phys. Lett. B **467**, 156 (1999)
4. A.V. Kisselev, V.A. Petrov, Z. Phys. C **75**, 277 (1997)
5. A.V. Kisselev, V.A. Petrov, Phys. At. Nucl. V. **60**, 1533 (1997)
6. J.C. Collins, Renormalization (Cambridge University Press, 1984)
7. J.C. Collins, F. Wilczek, A. Zee, Phys. Rev. D **18**, 242 (1978); J.C. Collins, Phys. Rev. D **58**, 094002 (1998)
8. A.V. Kisselev, V.A. Petrov, R.A. Ryutin, Yad. Fiz. V. **65**, 1 (2002)
9. ZEUS Collab., Abstract 853, Paper submitted to the 30th International Conference on High–Energy Physics (ICHEP2000) (Osaka, Japan, 2000)